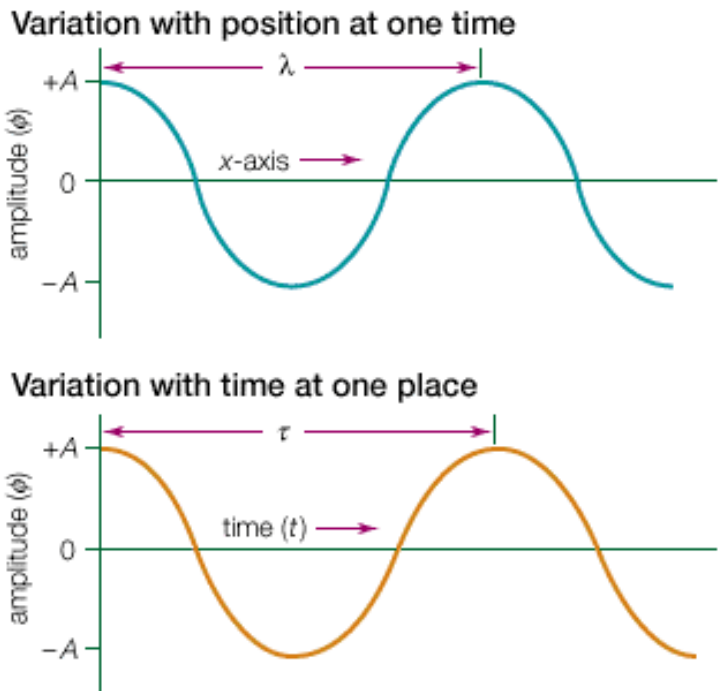
Quantum Mechanics-2

Wave velocity/phase velocity

- Wave motion is a form of disturbance which travels through medium due to the repeated motion of the particles of the medium about their mean positions, the motion being handed over from one particle to the next.
- Every particle begins its vibrations a little later than its predecessor and there is a progressive change of phase as we travel from one particle to the next.
- The phase relationship of these particles that we observe as a wave and the velocity with which the plane of equal phase travels through the medium, is known as **phase velocity.**
- Hence the velocity with which monochromatic wave propagates through medium is called the **wave velocity.**

The equation of motion for a plane progressive wave is given by

$y=A\cos(\omega t - kx)$ ……(1)

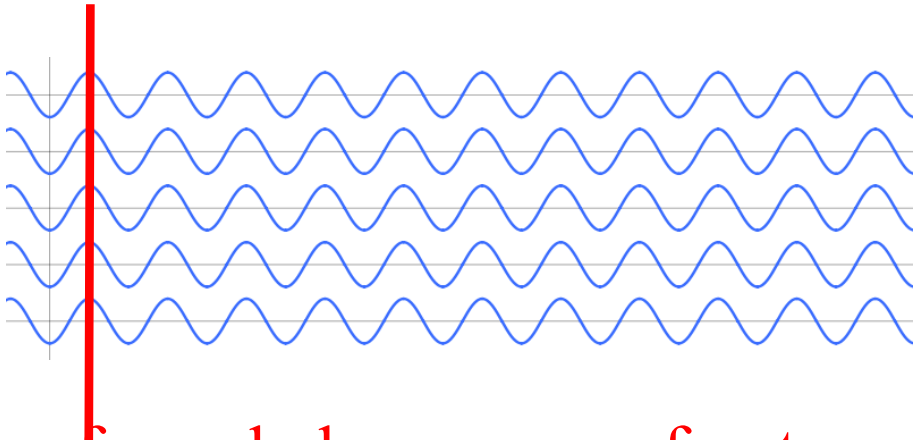


Where y= displacement of the wave depends upon space coordinate 'x' and time 't'. A is the amplitude and w is the angular frequency which is related to the frequency v by

$$\omega = 2\pi v$$

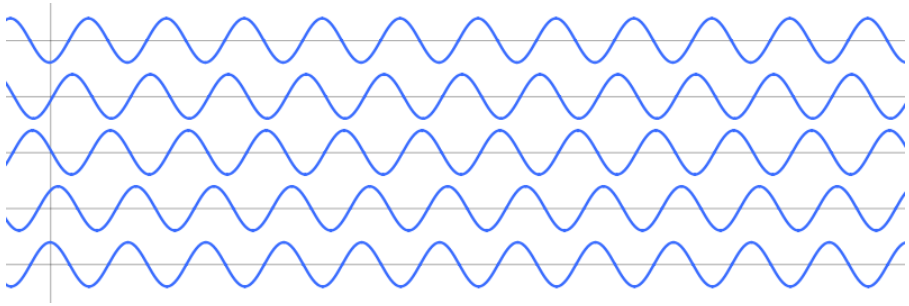
k is the phase constant and given by $k=2\pi/\lambda$

The argument of the cosine function represents the *phase* of the wave, ϕ, or the fraction of a complete cycle of the wave.



In-phase waves

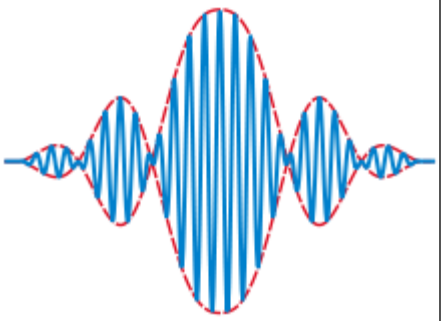
Line of equal phase = wavefront = contours of maximum field



Out-of-phase waves

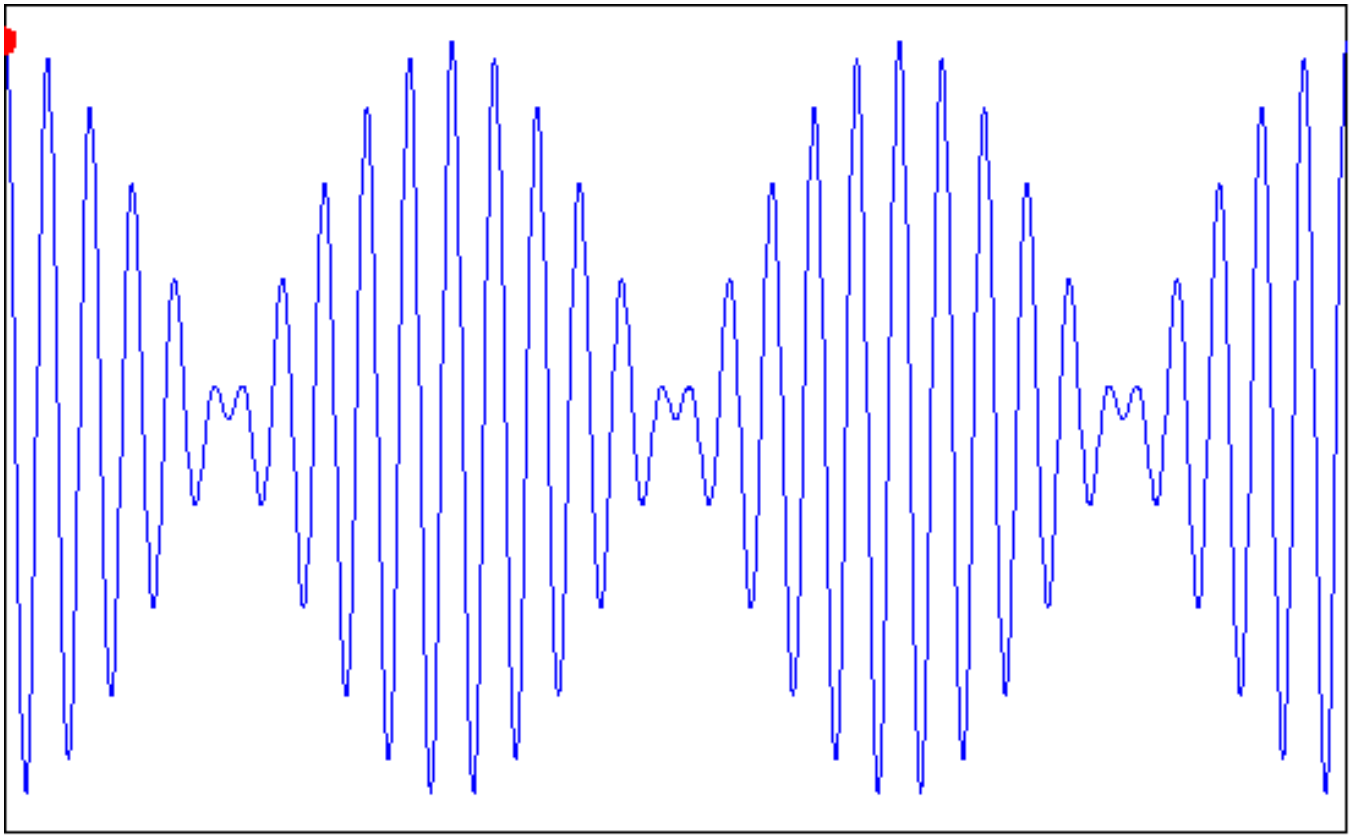
Group velocity: the concept of wave packet

- The group velocity of a wave is the velocity with which the overall shape of the waves' amplitudes known as the modulation or envelope of the wave propagates through space.
- For example, imagine what happens if a stone is thrown into the middle of a very still pond. When the stone hits the surface of the water, a circular pattern of waves appears. It soon turns into a circular ring of waves with a quiescent centre.



The ever expanding ring of waves is the wave group, within which one can discern individual wavelets of differing wavelengths travelling at different speeds. The longer waves travel faster than the group as a whole, but they die out as they approach the leading edge. The shorter waves travel more slowly and they die out as they emerge from the trailing boundary of the group.

The Group Velocity



This is the velocity at which the overall shape of the wave's amplitudes, or the wave 'envelope', propagates.

Expression for group velocity and wave velocity

Let the wave packet is formed by the superimposition of two waves of equal amplitude 'A' but of slightly different frequency *w* and *w*+*dw* and propagation constants k and k+dk. Displacement y_1 and y_2 of two waves

$y_1 = A \cos(wt-kx)$

$y_2 = A \cos[(w+dw)t - (k+dk)x]$

The resultant displacement $y = y_1 + y_2$

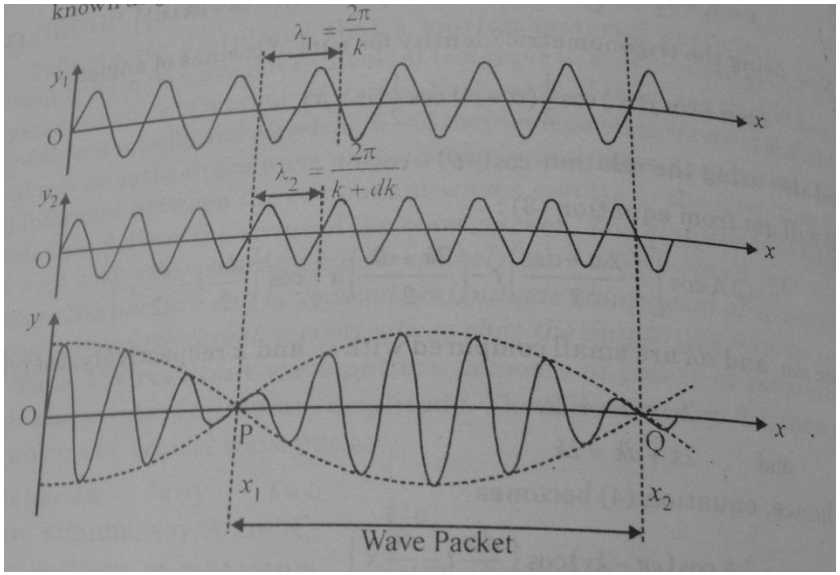
$y = A \{\cos(wt-kx)+\cos[(w+dw)t - (k+dk)x]\}$

$$y = 2A \cos\left[\left(\frac{2w+dw}{2}\right) t - \left(\frac{2k+dk}{2}\right) x\right] \cos\left(\frac{dw}{2} t - \frac{dk}{2} x\right)$$

Since *dw* and *dk* are small compared with *w* and *k*

$$y = 2A \cos[wt - kx] \cos\left(\frac{dw}{2} t - \frac{dk}{2} x\right)$$

This equation represent a wave of angular frequency *w* and wave number *k* which has been modulated by a wave of angular frequency *dw/2* and wave number *dk/2. The effect of modulation is to produce successive wave packets.*



Wave packet produced by superimposition of two waves with slightly different frequency

- The resultant wave has two parts:
  - The factor $\cos\,[wt - kx]$ represents carrier wave of phase velocity

$$v_p = \frac{w}{k} = \frac{2\pi\vartheta}{2\pi/\lambda} = \vartheta\lambda$$

  - The other factor $\cos(\frac{dw}{2}\,t - \frac{dk}{2}\,x)$ represents modulated wave

$$v_g = \frac{dw/2}{dk/2} = \frac{dw}{dk}$$

Relation between phase velocity and group velocity

- Phase velocity

$$v_p = \frac{w}{k} \text{ means, } w = v_p k$$

- Group velocity

$$v_g = \frac{dw}{dk} = \frac{d(v_p k)}{dk} = v_p + k\frac{dv_p}{dk}$$

$$v_g = v_p + k\frac{dv_p}{d\lambda}.\frac{d\lambda}{dk}$$

$$\frac{d\lambda}{dk} = -\frac{\lambda^2}{2\pi}$$

$$\text{Therefore, } v_g = v_p + k\frac{dv_p}{d\lambda}.\left(-\frac{\lambda^2}{2\pi}\right)$$

$$v_g = v_p - \lambda\frac{dv_p}{d\lambda}$$

phase velocity= wave velocity of carrier : $v_p = \frac{\omega}{k}$

group velocity= wave velocity of envelope : $v_g = \frac{\Delta\omega}{\Delta k}$

for more than two wave contiributions: $v_g = \frac{d\omega}{dk}$

Wave function
Postulates of Quantum Mechanics

Wave function

Quantum mechanics acknowledges the wave-particle duality of matter by supposing that, rather than traveling along a definite path, a particle is distributed through space like a wave. The wave that in quantum mechanics replaces the classical concept of particle trajectory is called a wavefunction, ψ ("psi").

A wave function in quantum mechanics describes the quantum state of an isolated system of one or more particles. There is *one* wave function containing all the information about the entire system, not a separate wave function for each particle in the system.

Physically acceptable wave function

The wave function and its first derivative must be:

(1) Finite; The wave function must be single valued. This means that for any given values of x and t , $\Psi(x,t)$ must have a unique value. This is a way of guaranteeing that there is only a single value for the probability of the system being in a given state.

(2) Square-integrable; The wave function must be square-integrable. In other words, the integral of $|\Psi|^2$ over all space must be finite. This is another way of saying that it must be possible to use $|\Psi|^2$ as a probability density, since any probability density must integrate over all space to give a value of 1, which is clearly not possible if the integral of $|\Psi|^2$ is infinite. One consequence of this proposal is that must tend to 0 for infinite distances.

(3) Continuous wave function; **A rapid change would mean that the derivative of the function was very large (either a very large positive or negative number). In the limit of a step function, this would imply an infinite derivative. Since the momentum of the system is found using the momentum operator, which is a first order derivative, this would imply an infinite momentum, which is not possible in a physically realistic system.**

All first-order derivatives of the wave function must be continuous. Following the same reasoning as in condition 3, a discontinuous first derivative would imply an infinite second derivative, and since the energy of the system is found using the second derivative, a discontinuous first derivative would imply an infinite energy, which again is not physically realistic.

Postulates of Quantum Mechanics

Postulate 1:

State and wave functions. Born interpretation

The state of a quantum mechanical system is completely specified by a wave function ψ (r, t) that depends on the coordinates of the particles (r) and time (t). These functions are called *wave functions or state functions.*

For 2 particle system:

$$\Psi = \Psi(x_1, y_1, z_1, x_2, y_2, z_2, t)$$

Wave function contains all the information about a system.

wave function ⇔ classical trajectory

(Quantum mechanics) (Newtonian mechanics)

If ψ_1 and ψ_2 represent two physically-realizable states of the system, then the linear combination

$$\psi = c_1\psi_1 + c_2\psi_2$$

where c_1 and c_2 are arbitrary complex constants, represents a third physically realizable state of the system.

Note:

Wave function $\psi(x, t)$ ➔ position and time probability amplitude

Quantum mechanics describes the outcome of an ensemble of measurements, where an ensemble of measurements consists of a very large number of identical experiments performed on identical non-interacting systems, all of which have been identically prepared so as to be in the same state.

Meaning of wave function:

$$P(r) = |\psi|^2 = \int \psi^* \psi d\tau$$

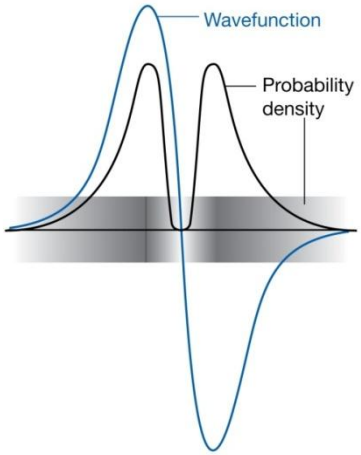
=> the probability that the particle can be found at a particular point x and a particular time t. (Born's / Copenhagen interpretation)

Implications of Born's Interpretation

(1) Positivity:

$$P(r) >= 0$$

The sign of a wave function has no direct physical significance: The positive and negative regions of this wave function both correspond to the same probability distribution.



(2) Normalization:

$$\int_{all_space} \Psi^* \Psi d\tau = 1$$

i.e. the probability of finding the particle in the universe is 1.

Postulate 2

To every physical property, observable in classical mechanics, there corresponds a linear, hermitian operator in quantum mechanics.

Every observable in quantum mechanics is represented by an operator which is used to obtain physical information about the observable from the state function. For an observable that is represented in classical physics by a function $Q(x,p)$, the corresponding operator is $Q(\hat{x}, \hat{p})$.

Observable	Operator
Position	$\hat{x}$
Momentum	$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$
Energy	$E = \frac{\hat{p}^2}{2m} + V(\hat{x}) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$

Postulate 3

In any measurement of the observable associated with the operator , the only values that will ever be observed are the eigen values 'a' which satisfy the eigen value equation:

$$\hat{A}\Psi = a\Psi$$

This is the postulate that the values of dynamical variables are quantized in quantum mechanics.

Eigen Function and Eigen value

$$\hat{A}\, f(x) = kf(x)$$

f(x) is eigenfunction of $\hat{A}$ with eigen value k

Postulate 4

For a system in a state described by a normalized wave function ψ, the average or expectation value of the observable corresponding to A is given by:

$$\langle A \rangle = \int \Psi^* \hat{A} \Psi \, d\tau$$

Mean value theorem

Expectation value in general:

$$\langle a \rangle = \frac{\int_{-\infty}^{\infty} \Psi^*(x) \hat{A} \Psi(x) dx}{\int_{-\infty}^{\infty} \Psi^*(x) \Psi(x) dx} \qquad = \int_{-\infty}^{\infty} \Psi^*(x) \hat{A} \Psi(x) dx \quad \text{(if normalized)}$$

The fourth postulates states what will be measured when large number of identical systems are interrogated one time. Only after large number of measurements will it converge to <a>.

In QM, the act of the measurement causes the system to "collapse" into a single eigen state and in the absence of an external perturbation it will remain in that eigenstate.

Postulate 5

The wave function of a system evolves in time in accordance with the time dependent Schrodinger equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\,\Psi$$

Postulates of Quantum Mechanics

Summary

Postulate 1:

The state of a quantum mechanical system is completely specified by a wave function ψ (r, t) that depends on the coordinates of the particles (r) and time (t). These functions are called *wave functions or state functions.* Wave function contains all the information about a system.

Postulate 2:

To every physical property, observable in classical mechanics, there corresponds a linear, hermitian operator in quantum mechanics.

Postulate 3:

In any measurement of the observable associated with the operator, the only values that will ever be observed are the eigen values 'a' which satisfy the eigen value equation: $\hat{A}\Psi = a\Psi$

Postulate 4:

For a system in a state described by a normalized wave function ψ, the average or expectation value of the observable corresponding to A is given by: $\langle A \rangle = \int \Psi^* \hat{A} \Psi \, d\tau$

Postulate 5:

The wave function of a system evolves in time in accordance with the time dependent Schrodinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

Schrödinger's Wave Equation (Derivation)

Time Dependent Schrödinger's Wave Equation

Considering a complex plane wave:

$$\Psi(x,t) = Ae^{i(kx-\omega t)} \qquad \text{..... (1)}$$

Using k= 2π/λ, $\boldsymbol{\omega = 2\pi\nu}$ and v= $\boldsymbol{\nu\lambda}$ in equation (1) and rearranging it we get

$$\boldsymbol{\psi = A\, e^{-i2\pi\nu}}(t - x/\nu\lambda)$$

OR $$\boldsymbol{\psi = A\, e^{-i2\pi}}(t\nu - x/\lambda)$$

$$\boldsymbol{\psi = A\, e}\exp\left(-i2\pi\left(\frac{th\nu}{h} - \frac{xh}{\lambda h}\right)\right)$$

$$\boldsymbol{\psi = A\, e}\exp\left(-i2\pi\left(\frac{tE}{h} - \frac{xP}{h}\right)\right)$$

Where E = h$\boldsymbol{\nu}$ is the energy, P = h/λ

OR $\psi = A\, e \exp\left(-i\frac{2\pi}{h}(tE - xP)\right)$

$$\psi = A\, e \exp\left(-\frac{i}{\hbar}(tE - xP)\right)$$

OR $\psi = A\, e^{(-i/\hbar(tE - xP))}$ ..... (3)

Where $\hbar = h/2\pi$

Eq. (3) Wave equivalent of unrestricted particle of the total energy E and momentum P of particle along + X axis

This expression eq. (3) for wave function is correct only for moving particles while we are interested in situation for motion of particle under various restrictions.

Let us now obtain a fundamental differential equation for Ψ which can be solved under various specific restriction.

Differenting equation (3) twice with x gives

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-P^2}{\hbar^2} \psi$$

OR

$$P^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \quad \text{..... (4)}$$

Differential equation (3) once with t gives

$$\frac{\partial \psi}{\partial t} = \frac{-E\psi}{\hbar}$$

OR

$$E\psi = \frac{-\hbar}{i} \frac{\partial \psi}{\partial t} \quad \text{..... (5)}$$

At a speed less than velocity of light, total energy E of particle is

$$E = \frac{p^2}{2m} + V(x) \qquad \text{..... (6)}$$

Where V(x) potential; energy and is a function of position

Multiplying eq. no. (6) by ψ on both sides

$$E\psi = \frac{p^2}{2m}\psi + V\,\psi \qquad \text{..... (7)}$$

From eq. (4, (5) and (6), time dependent form of Schrodinger's equation is

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

OR $$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)\psi$$ ..... (8)

OR $$E\,\psi = H\,\psi$$ ..... (9)

Where, $E = i\hbar\frac{\partial\psi}{\partial t}$ and $H = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)$

Called energy and Hamiltonian operator respectively.

Equation (8) and (9) gives time dependent form of Schrodinger's equation.

Time Independent Schrödinger's Wave Equation

The time-dependent Schrodinger equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi. \qquad \text{..... (1)}$$

Assume that V is independent of time t. In that case the Schrodinger equation can be solved by the method of separation of variables:

$$\Psi(x,t) = \psi(x)\, f(t)$$

Then on differentiation we get

$$\frac{\partial \Psi}{\partial t} = \psi \frac{df}{dt}, \quad \frac{\partial^2 \Psi}{\partial x^2} = \frac{d^2\psi}{dx^2} f \qquad \text{..... (2)}$$

From eq. (1) And (2) we get

$$i\hbar\psi \frac{df}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} f + V\psi f. \qquad \text{..... (3)}$$

Dividing eq. (3) through by ψf, we get

$$i\hbar \frac{1}{f}\frac{df}{dt} = -\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V. \qquad \text{..... (4)}$$

Now the left side is a function of t alone, and the right side is a function of x alone. The only way this can possibly be true is if both sides are constant (say E) Then,

$$i\hbar \frac{1}{f}\frac{df}{dt} = E. \qquad \text{..... (5)}$$

Which gives

$$f(t) = e^{-iEt/\hbar}.$$

From eq. (4)and (5)

$$-\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{d^2\psi}{dx^2} + V = E \qquad \text{..... (6)}$$

or

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+V\psi=E\psi.$$

$$-\frac{\partial^2\psi}{\partial x^2}=\frac{2m}{\hbar^2}(E-V)\psi$$

or

$$\frac{\partial^2\psi}{\partial x^2}+\frac{2m}{\hbar^2}(E-V)\psi=0$$

Which is called the time-independent Schrodinger equation; it requires the knowledge of the potential V. Before solving the time-independent Schrodinger equation for certain potentials.

Application of Schrödinger's equation to One Dimensional Problem

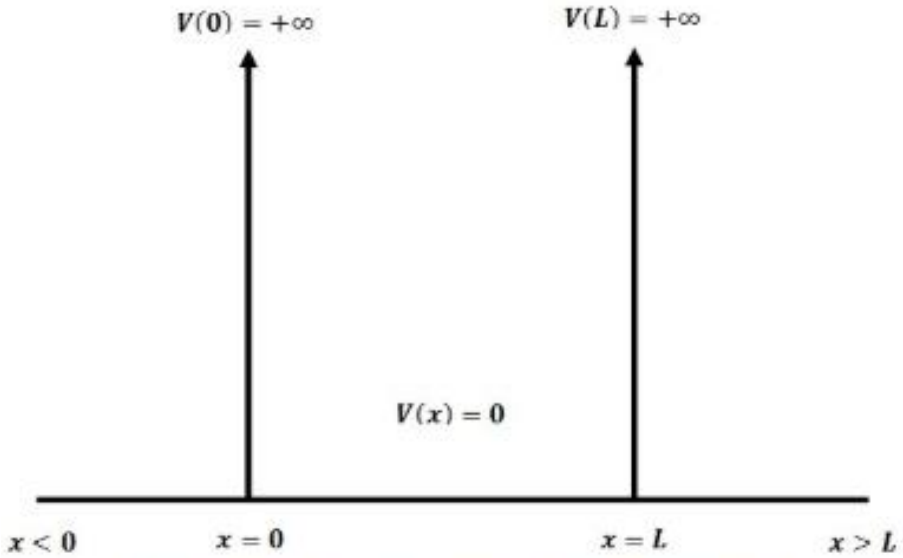
Particle in a box

- The particle in a box problem is a common application of a quantum mechanical model to a simplified system consisting of a particle moving horizontally within an infinitely deep well from which it cannot escape.
- The solutions to the problem give possible values of E and ψ that the particle can possess.
- E represents allowed energy values and $\psi(x)$ is a wave-function, which when squared gives us the probability of locating the particle at a certain position within the box at a given energy level.

Consider a particle with mass m and mechanical energy E in an environment characterized by potential energy function V(x) .

We confine the particle to a region between x = 0 and x = L

Let us write the potential (the potential of infinite depth) as



Particle in One-Dimensional Box(Infinite Potential Well)

$$V(x) = 0 \quad \text{for} \quad 0 \leq x \leq L$$
$$= \infty \quad \text{otherwise}$$

- **The potential energy is 0 inside the box (V=0 for 0<x<L) and goes to infinity at the walls of the box (V=∞ for x<0 or x>L).**
- **We assume the walls have infinite potential energy to ensure that the particle has zero probability of being at the walls or outside the box.**
- **This is necessary to apply the proper boundary conditions while solving the Time Independent Schrödinger Equation (TISE) for infinitely deep square well.**

Conditions the wave function must obey are

1. $\psi(x)$ and $\psi'(x)$ are continuous functions.
2. $\psi(x) = 0$ if x is in a region where it is physically impossible for the particle to be.
3. $\psi(x) \to 0$ as $x \to +\infty$ and $x \to -\infty$.
4. $\psi(x)$ is a normalized function.

The Time-independent Schrödinger equation (TISE) for a particle of mass m moving in one direction with energy E is

$$-\frac{\partial^2\psi}{\partial x^2} = \frac{2m}{\hbar^2}(E - V)\,\psi$$

Or

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

- **This equation can be modified for a particle of mass m free to move parallel to the x-axis with zero potential energy (V = 0 everywhere)**
- **Out side the box the solution is ZERO i.e. ψ=0**
- **Inside the box the TISE reduces to**

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}E\psi = 0$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$$ Where $$k^2 = \frac{2m}{\hbar^2} E$$

This equation has well known solution as

$$\psi = A \sin kx + B \cos kx$$

Where A and B are arbitrary constants. Typically, these constants are fixed by the boundary conditions of the problem.

Both Ψ and dΨ/dx are continuous.

Boundary conditions, the probability of finding the particle at x=0 or x=L is zero implies (ψ(x)=0) .

When x=0 , then sin (0)=0 and cos (0)=1 ; therefore

$$\psi(0) = A\sin(0) + B\cos(0) = 0 \Rightarrow B = 0$$

Then for x=L, the following is true

$$\psi(L) = A\sin(kL) = 0 \Rightarrow kL = 0, \pi, 2\pi, 3\pi, \ldots\ldots n\pi$$

$$\therefore kL = n\pi \,(\forall n = 1,2,3,\ldots)$$

But k=0 is no good. Again, that would imply ψ(x)=0. The negative solutions give nothing new, since sin(-θ)=-sin(θ) and the minus sign can be absorbed into A. So the distinct solutions are

$$\psi_n(x) = A\sin k_n x \quad \text{for} \quad k_n = \frac{n\pi}{a}, \quad \text{with } n = 1,\ 2,\ 3, \ldots.$$

Accordingly, for the possible quantum states of the particle in the box are the wave function can take the forms:

For $0 \le x \le L$:

$$\psi_n(x) = A\sin\left(\frac{n\pi}{L}x\right), \quad \text{where} \quad n = 1,2,3,\ldots\ , \text{ and}$$

$$\psi_n(x) = 0 \quad \text{for all} \quad x < 0 \ \text{ and } x > L.$$

Through normalization, A can be fixed. Which implies that probability of finding the particle inside box is one i.e

$$\int_0^L |\psi_n(x)|^2 dx = 1$$

Let's carry out the calculations:

$$\int_0^L |\psi_n(x)|^2 dx = \int_0^L A^2 \sin^2\left(\frac{n\pi}{L}x\right) dx$$

Let's use a "dummy variable" $y \equiv \frac{n\pi}{L}x$

then $x = \frac{L}{n\pi}y$ and $dx = \frac{L}{n\pi}dy$

The lower limit is still 0, but the upper limit has to be changed to:

$$y_{\text{upper_limit}} = \frac{n\pi}{L}x_{\text{upper_limit}} = \frac{n\pi}{L}L = n\pi$$

$$= A^2 \frac{L}{n\pi}\int_0^{n\pi} \sin^2(y)dy$$

$$\int_0^{n\pi} \sin^2(y)dy = \frac{n\pi}{2}$$

So, we can continue from the preceding slide:

$$= A^2 \frac{L}{n\pi} \times \frac{n\pi}{2} = \frac{A^2 L}{2} = 1$$

From which we obtain immediately:

$$A = \sqrt{\frac{2}{L}}$$

and the complete solution for $\psi(x)$:

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$$

Energy quantization:

The wavenumber k is related to the energy as :

$$k = \sqrt{\frac{2mE}{\hbar^2}}, \quad \text{so} \quad E = \frac{\hbar^2 k^2}{2m}$$

But, as we have found, the allowed values of k are : $k_n = \frac{n\pi}{L}$

Combining these two

$$\mathbf{E_n = \frac{n^2 h^2}{8mL^2}} \quad , \quad \mathbf{where\ n = 1, 2, 3....}$$

Thus from all discussion done above. The normalized wave-functions and allowed energies for a particle 1-dimensional box is given by

$$\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x$$

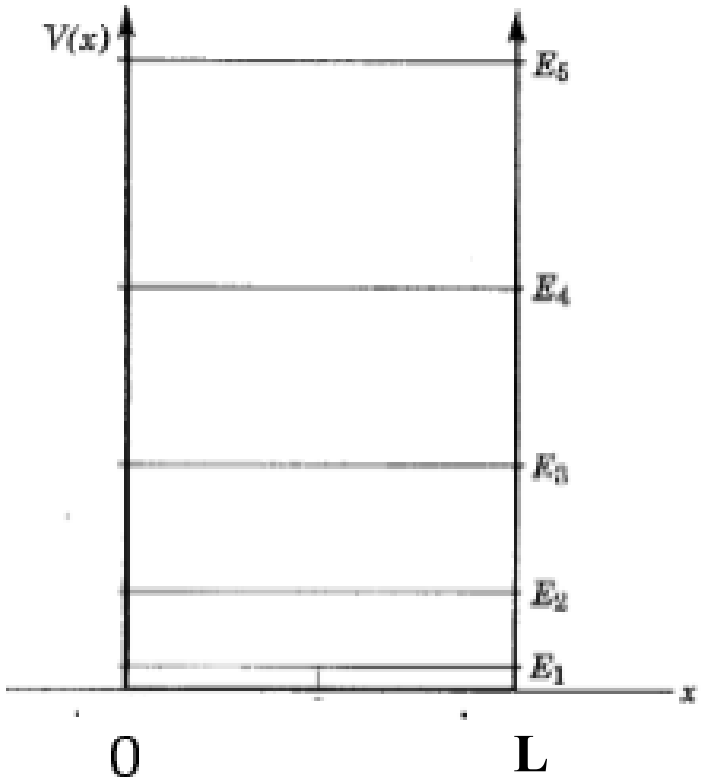
$$\mathbf{E_n = \frac{n^2 h^2}{8mL^2}}$$

•A quantum particle in the infinite square well can only have certain specific values of energies. The total energy of the particle in the well is quantized.

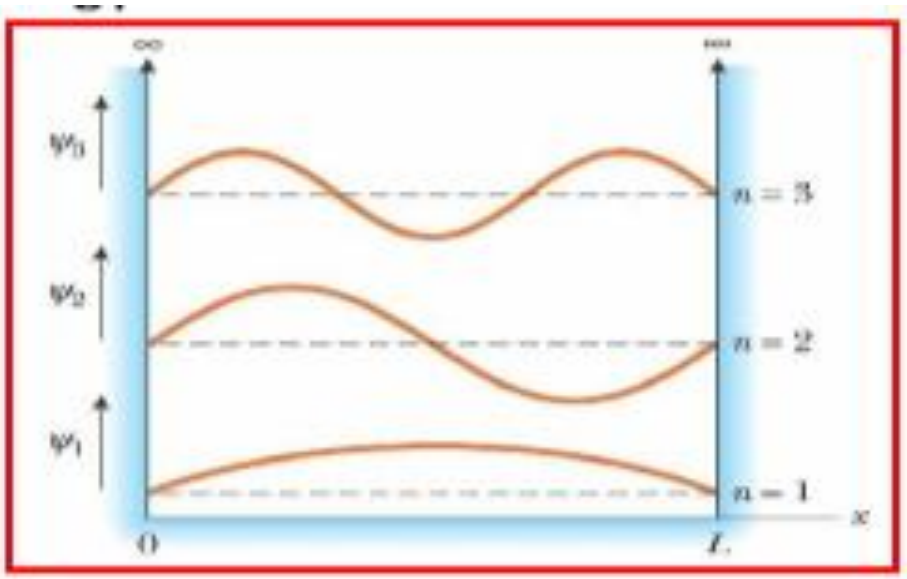
•The lowest possible energy of a particle is NOT zero.

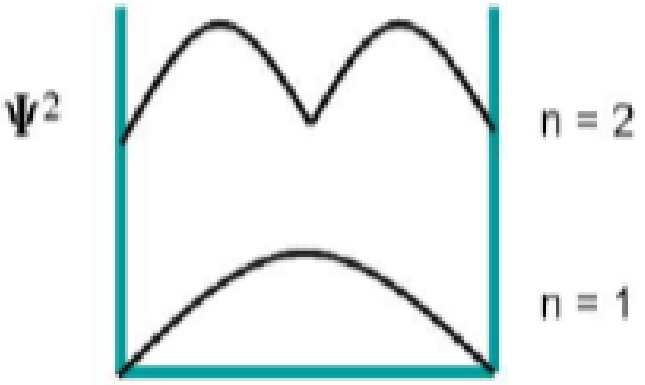
$$E_0 = \frac{\hbar^2 \pi^2}{2mL^2} = \frac{h^2}{8mL^2} \ (for \ n = 1)$$

This is called the zero-point energy (ground state energy) and means the particle can never be at rest because it always has some kinetic energy.



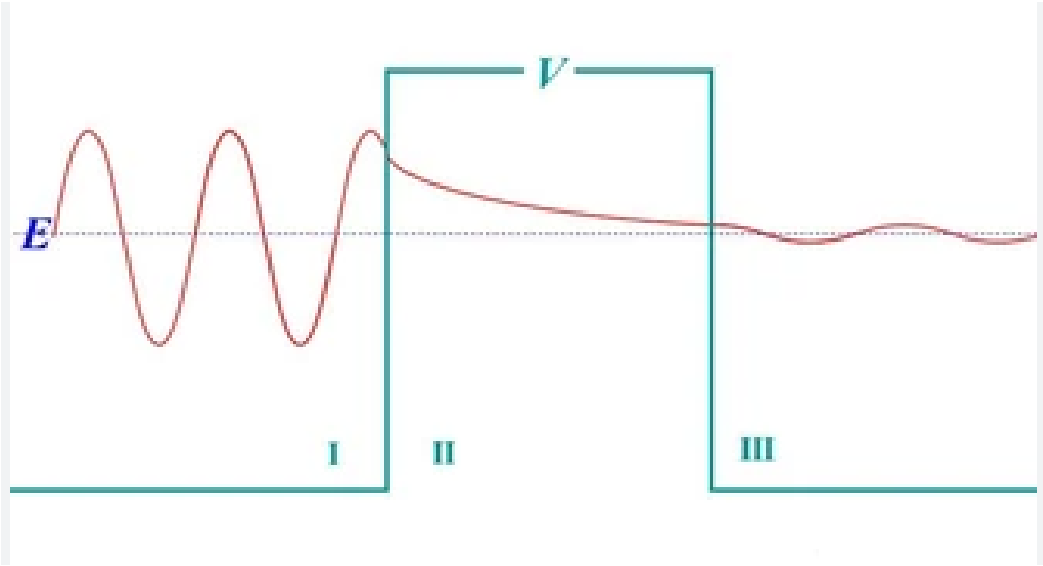
- This is also consistent with the Heisenberg Uncertainty Principle: if the particle had zero energy, we would know where it was in both space and time.
- The wave-functions for a particle in a box at the n=1, n=2 and n=3 energy levels look like as figure.
- The probability of finding a particle a certain spot in the box is determined by Squaring ψ .
- The probability distribution for a particle in a box at the n=1 and n=2 energy levels looks like as given in figure.





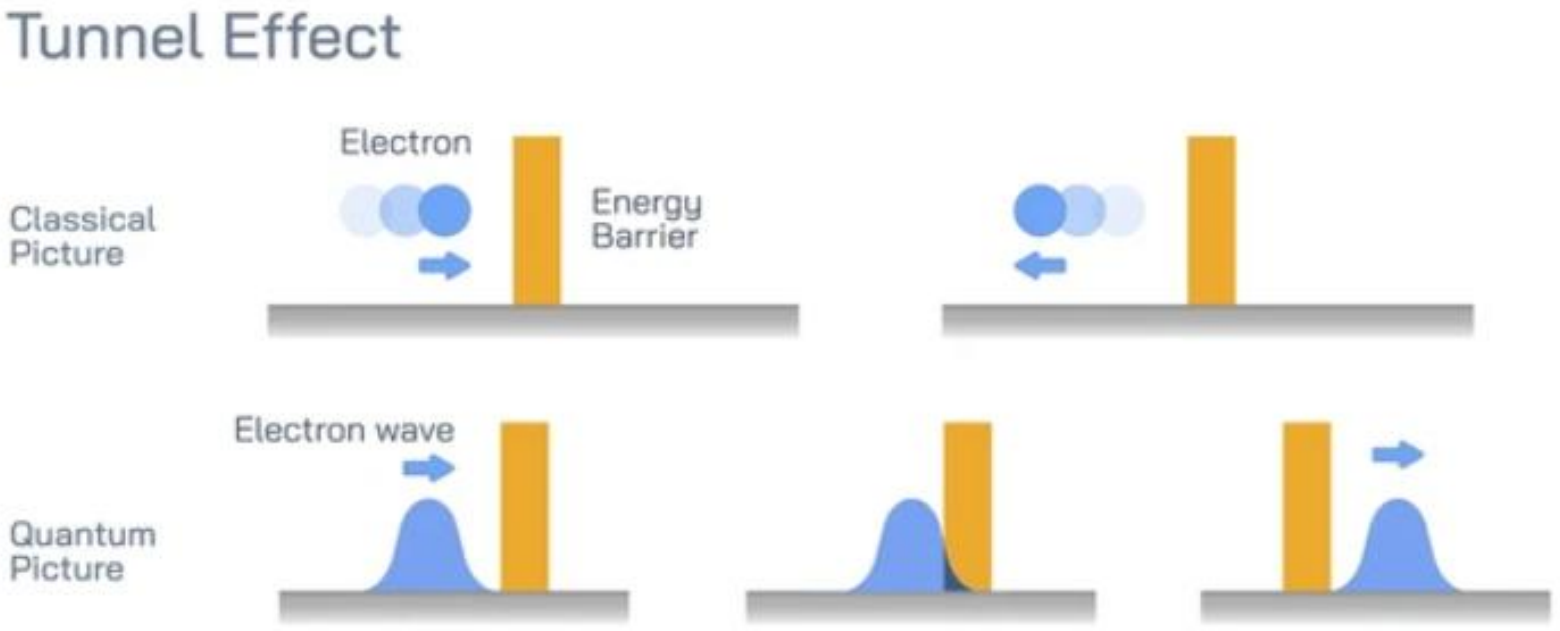
BARRIER PENETRATION AND QUANTUM TUNNELING.

➢According to classical ideas, a particle striking a hard wall has no chance of leaking through it. But, the behavior of a quantum particle is different due to the wave nature associated with it.



➢We know that when an electromagnetic wave strikes at the interface of two media, it is partly reflected and partly transmitted through the interface and enters the second medium.

➢In a similar way the de Broglie wave also has a - In a similar way the de Broglie possibility of getting partly reflected from the boundary of the potential well and partly penetrating through the barrier.



➢**Fig. shows a particle with energy E < V approaching potential barrier of height.**

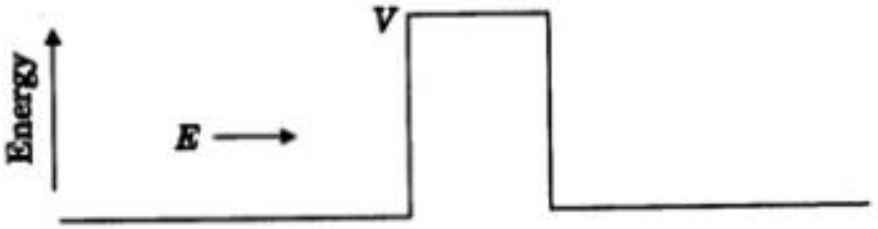
➢**An electron of total energy E approaches the barrier from the left. From the viewpoint of classical physics, the electron would be reflected from the barrier because its energy E is less than V.**

➢**For the particle to overcome the potential barrier, it must have an energy equal to or greater than V.**

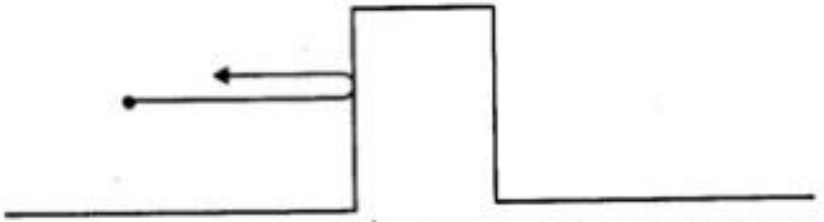
➢Quantum mechanics leads to an entirely new result. It shows that there is a finite chance for the electron to If shat the other side of the barrier.

➢It is noted that the electron tunneled through the potential barrier and hence in quantum mechanics, this phenomenon is called tunneling.

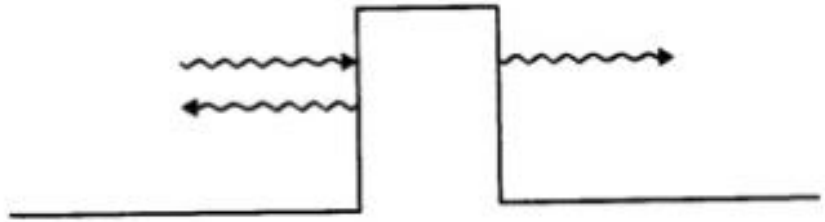
➢The transmission of electrons through the barrier is known as barrier penetration.



(a) A particle with energy E < V approaching a potential barrier.



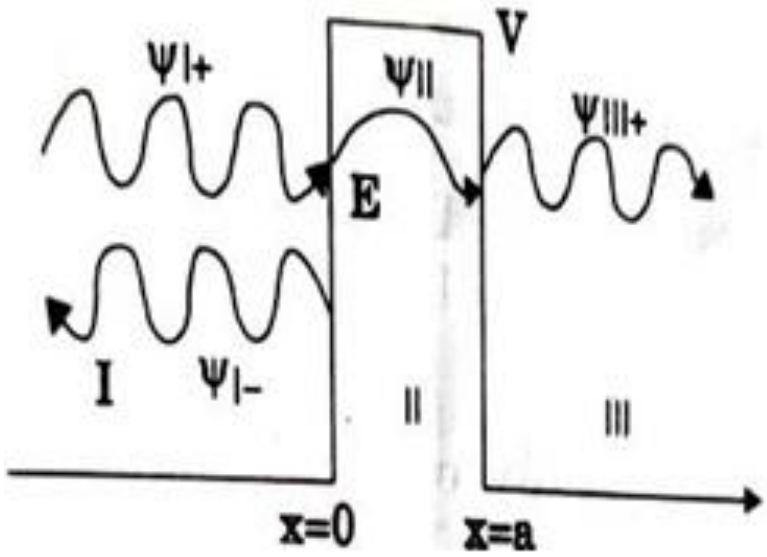
(b) From classical mechanics, the particle must be reflected by the barrier.



(c) In quantum mechanics, the de Broglie waves that represent the particle are partly reflected and partly transmitted, i.e., the particle has finite probability of penetrating the barrier

Expression for Transmission Probability

- Now let us consider the case of a particle of energy E < V approaching a potential barrier of finite height and width as shown in fig.
- The particle in region has certain probability of passing through the barrier to reach region II and then emerge out on the other side in region III.
- The particle lacks the energy to go over the top of the barrier, but tunnels through it. Higher the barrier and wider it is, the lesser is the probability of the particle tunneling through it.



➢Fig. shows When a particle of energy E<V , approaches a potential barrier, the de Broglie waves that correspond to the particle are particle are partly reflected and partly transmitted. That is the particle has a finite chance of penetrating the barrier.

➢Let us now consider a beam of identical particles, all having kinetic energy . The beam is incident on the potential barrier of height and width a from region.

➢On both sides of the barrier V = 0. This means that no forces act on particles in regions I and III.

The wave function Ψ_{II} represents the particle inside the barrier. Some of the particles end up in region III while the others return to region I.

Quantum mechanics shows that the transmission probability for a particle to pass through the barrier is given by

$$T = \frac{\text{Number of particles transmitted}}{\text{Number of particles incident}}$$

This probability is approximately given by

$$T = T_o e^{-2ka}$$

Where $K = \frac{\sqrt{2m(V-E)}}{h}$ and 'a' is the width of the barrier.

T_o is a constant close to unity. It shows that the probability of particle penetration through a potential barrier depends on the height and width of the barrier

Significance of the study of barrier penetration problems

- Tunneling is a very important physical phenomenon which occurs in certain semiconductor diodes. In such diodes electrons pass through potential barriers even though their kinetic energies are smaller than the barrier heights.
- The tunneling effect also occurs in the case of the alpha particles. The kinetic energy of alpha particle is only a few MeV but it is able to escape from a nucleus whose potential wall is perhaps 25MeV high.
- The ability of electrons to tunnel through a potential barrier is used in the Scanning Tunneling Microscope (STM) to study surfaces on an atomic scale of size.