MAIC 101: DCDE Assignment 1

Instructions

1. Answer all questions and all questions carry equal marks.
2. Write answers on A4 papers and spiral / staple all the sheets in the sequential order.
3. Class Representative shall collect assignment copies (in hard copy) from all the students and submit it to the concerned course teacher on or before **September 25, 2026 5:00 PM**.
4. Answer sheets received after due date shall summarily be rejected.
5. Students who submit answer sheets belonging to others will receive a marks of zero, and such actions will be deemed unfair, leading to appropriate consequences.

Matrix Theory

1. Show that the vectors $\mathbf{X_1} = \begin{pmatrix} 2 \\ 4 \\ 0 \end{pmatrix}$, $\mathbf{X_2} = \begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$, $\mathbf{X_3} = \begin{pmatrix} -1 \\ 2 \\ -3 \end{pmatrix}$ are linearly independent.
2. Prove that every matrix can be represented as a sum of symmetric and skew-symmetric matrices.
3. Using elementary row operations, reduce the matrix $\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$ to reduced-row echelon form.
4. Find the rank of a matrix $\begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 4 & 2 \\ 0 & 2 & 2 & 1 \end{pmatrix}$.
5. Check whether the linear system of equations $2x + 2y - z = 1, 4x + 2z = 2, 6y - 3z = 4$ consistent? Does it have a unique solution?
6. Check the consistency of the system of equations $10x+4y-2z = 14$, $-3w-15x+y+2z = 0$, $w + x + y = 6$, $8w - x + 5y - 10z = 26$ and find the solution if consistent.
7. Find the values of k for which the system of equations $3x + y - kz = 0, 4x - 2y - 3z = 0, kx + 4y + kz = 0$ has a non-trivial solution.
8. Investigate for what values of λ and μ, the system of equations $x+y+z = 6, x+2y+3z = 10, x + 2y + \lambda z = \mu$ have (i) no solution, (ii) unique solution, and (iii) infinitely many solutions?
9. Find the eigenvalues and corresponding eigenvectors of the matrix $\begin{pmatrix} 1 & 0 & 12 \\ 2 & -5 & 0 \\ 1 & 0 & 2 \end{pmatrix}$.

10. If possible, diagonalize the matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix}$. If diagonalizable, find A^8.

11. If possible, diagonalize the matrix $A = \begin{pmatrix} 2 & 4 & 6 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{pmatrix}$.

12. Reduce the quadratic form $3x^2 + 3z^2 + 4xy + 8xz + 8yz$ to its canonical form. Specify the orthogonal matrix of transformation. Find the nature, index and signature of the quadratic form.

13. Reduce the quadratic form $2x^2 + 5y^2 - 6z^2 - 2xy + 8xz - yz$ to its canonical form. Specify the orthogonal matrix of transformation. Find the nature, index and signature of the quadratic form.