Assignment

(7) $v_p = \frac{\omega}{k}$ $\quad v_g = \frac{d\omega}{dk}$

$\boxed{\omega = k v_p}$ $\quad v_g = \frac{d}{dk}(k v_p)$

$v_g = v_p + k\frac{dv_p}{dk}$

$k = \frac{2\pi}{\lambda}$

$dk = -\frac{2\pi}{\lambda^2} d\lambda$

$\frac{k}{dk} = -\frac{\lambda}{d\lambda}$

$\boxed{v_g = v_p - \frac{\lambda dv_p}{d\lambda}}$ $\quad \underline{v_g < v_p}$ for dispersive medium

(8) $E = h\nu$ or $\nu = \frac{E}{h}$

from $E = mc^2$

$\boxed{\nu = \frac{mc^2}{h}}$ and $\boxed{\lambda = \frac{h}{mv}}$

$v_p = \nu\lambda = \frac{mc^2}{h} \cdot \frac{h}{mv}$

$\boxed{v_p = \frac{c^2}{v}}$ means $\boxed{v_p > c}$

(9) $v_p = v_d$ & $d = \frac{h}{mv_g}$

$E = \frac{1}{2} m v_g^2$

$E = h\nu$

So, $\nu = \frac{E}{h} = \frac{\frac{1}{2} m v_g^2}{h}$

So, $v_p = \frac{\frac{1}{2} m v_g^2}{h} \cdot \frac{h}{mv_g}$

$\boxed{v_p = \frac{1}{2} v_g}$

(10) We know that

$\omega = 2\pi\nu$ & $d\omega = 2\pi\, d\nu$

$k = \frac{2\pi}{\lambda}$ & $dk = -\frac{2\pi}{\lambda^2} d\lambda$

So $v_g = \frac{d\omega}{dk} = \frac{2\pi\, d\nu}{-\left(\frac{2\pi}{\lambda^2}\right) d\lambda} = -\lambda^2 \frac{d\nu}{d\lambda}$

(11) $E = h\nu$ $\quad \lambda = 1\,A^\circ = 10^{-10}\,m$

$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34}\,[J\cdot s] \times 3\times 10^{8}\,[m/s]}{10^{-10}\,[m]}\,[J]$

$= \frac{6.62 \times 10^{-34}\,[J\cdot s] \times 3\times 10^{8}\,[m/s]}{10^{-10}\,[m] \times 1.6 \times 10^{-19}\,[C]}\,[eV]$

$\boxed{E = 1.24 \times 10^{4}\,[eV]}$

(12) K.E $= 10$ eV

$\lambda = \frac{h}{\sqrt{2mE}} = 3.$

$= \frac{6.62 \times 10^{-34}\,[J\cdot s]}{\sqrt{2 \times 9.1 \times 10^{-31}\,[kg] \times 10 \times 1.6 \times 10^{-19}\,[J]}}$

$\boxed{\lambda = 3.87\,A^\circ}$

(13) $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$ here $E = qV$ joules

$q \rightarrow$ charge on α-particle $\quad V \rightarrow$ Volts

for α-particle $q = 2e$

$\therefore E = 2eV$

Thus $\lambda = \frac{h}{\sqrt{2m \times 2eV}} = \frac{h}{\sqrt{4meV}}$

m = mass of α-particle $= 4 \times$ mass of proton

$m = 4 \times 1.67 \times 10^{-27}$ kg

$e = 1.6 \times 10^{-19}$ coulomb

$h = 6.62 \times 10^{-34}$ J-s

$V = 200$ Volts

$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{4 \times 4 \times 1.67 \times 10^{-27} \times 1.6 \times 10^{-19} \times 200}}$

$\lambda = \frac{6.62 \times 10^{-34}}{\sqrt{[29.241 \times 10^{-46}}}$

$\boxed{\lambda = 2.26 \times 10^{-2}\,A^\circ}$