EMT

MAXWELL'S EQUATIONS

&

ELECTROMAGNETIC WAVES

Gradient, Divergence and Curl

Gradient, divergence and curl", commonly called "grad, div and curl", refer to a very widely used family of differential operators.

The Del Operator

$$\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

The ***gradient*** of a scalar-valued function $f(x, y, z)$ is the vector field

$$\text{grad}\, f = \nabla f = \frac{\partial f}{\partial x}\hat{\mathbf{i}} + \frac{\partial f}{\partial y}\hat{\mathbf{j}} + \frac{\partial f}{\partial z}\hat{\mathbf{k}}$$

Note that the input, f, for the gradient is a scalar-valued function, while the output, ∇f, is a vector-valued function.

The ***divergence*** of a vector field $\mathbf{F}(x, y, z)$ is the scalar-valued function

$$\text{div}\, \mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

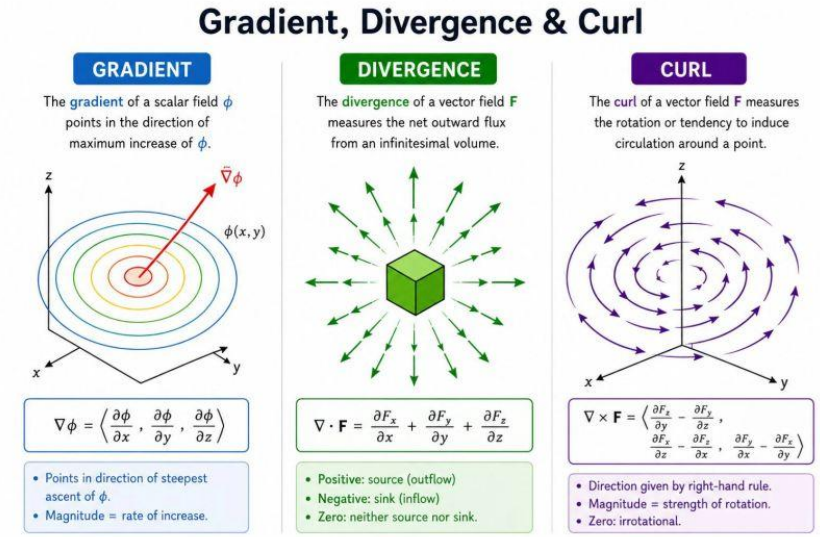
Note that the input, $\mathbf{F}$, for the divergence is a vector-valued function, while the output, $\nabla \cdot \mathbf{F}$, is a scalar-valued function.

The ***curl*** of a vector field $\mathbf{F}(x, y, z)$ is the vector field

$$\text{curl}\, \mathbf{F} = \nabla \times \mathbf{F} = \left(\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}\right)\hat{\mathbf{i}} - \left(\frac{\partial F_3}{\partial x} - \frac{\partial F_1}{\partial z}\right)\hat{\mathbf{j}} + \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)\hat{\mathbf{k}}$$

Note that the input, $\mathbf{F}$, for the curl is a vector-valued function, and the output, $\nabla \times \mathbf{F}$, is a again a vector-valued function.

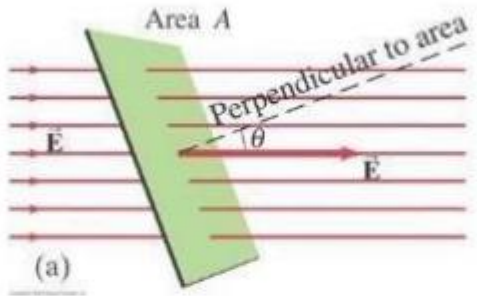
Gradient takes in a scalar function and outputs vector field Points in the direction of the steepest increase



Two key concepts in vector calculus are divergence and curl, the latter of which is sometimes called circulation. Basically, divergence has to do with how a vector field changes its magnitude in the neighborhood of a point, and curl has to do with how its direction changes.

Electric flux

- The electric flux refers to the electric field lines that penetrate a given surface

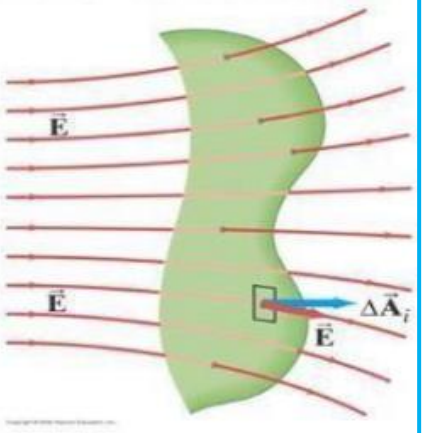


- For a given uniform electric field *E* passing thru an area *A*, the electric flux is defined as:

$$\Phi_E = EA\cos\theta$$

Flux For a Closed Area

- In general, an electric field is *not* uniform and the area is *not* flat !



- Any area can be divided up into *n* number of small elements of surface area ΔA, sufficiently small to be considered flat and the E-field is rather invariant over this small area.

- Then flux is given by: $\Phi_E = \oint \vec{E} \bullet dA$

Gauss law of electric fields

The total of the electric flux out of a closed surface is equal to the charge enclosed divided by the permittivity

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} Q_{\text{enc}},$$

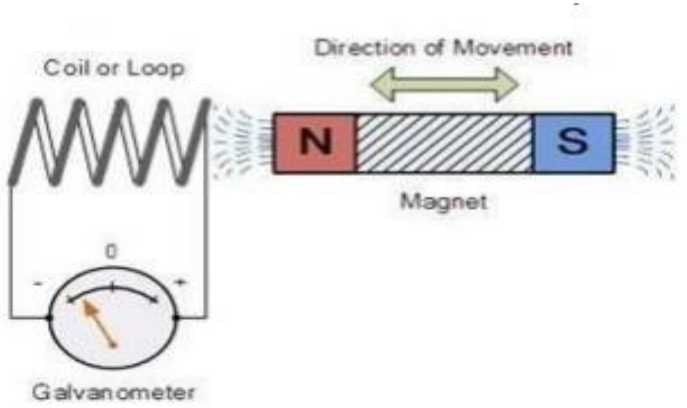
Gauss law of Magnetism

- The net magnetic flux out of any closed surface is zero.
- For any closed surface ,the magnetic flux directed inward toward the south pole will equal the flux outward from the north pole.

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's Law

Any change in the magnetic field of a coil of wire will cause an EMF to be induced in the coil. This EMF induced is called induced EMF and if the conductor circuit is closed the current will also, circulate through the circuit and this current is called induced current.



$$\mathcal{E} = -\frac{d\Phi}{dt}$$

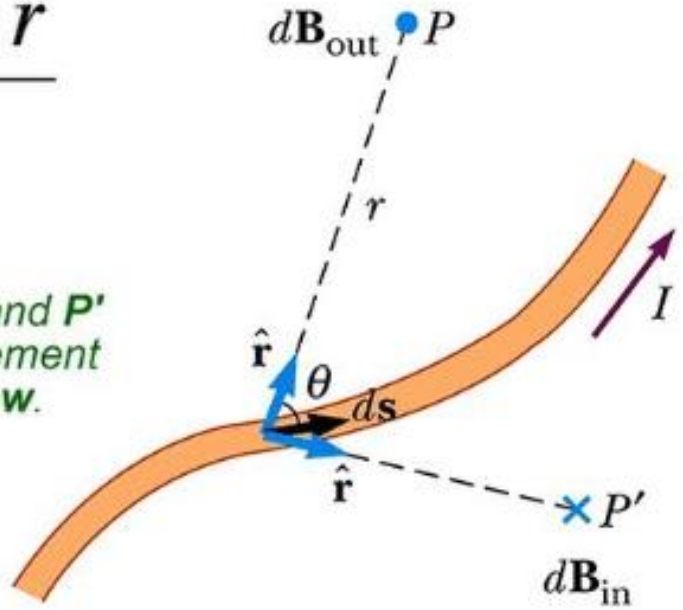
$$\oint \vec{E} \cdot \vec{dl} = -\frac{d\Phi_B}{dt}$$

The Biot-Savart Law

The Biot-Savart Law

The Magnetic Field produced by the current in the wire

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$



*The magnetic field d**B** at a point **P** and **P'** due to the current **I** thru a length element d**s** is given by the **Biot-Savart law**.*

This Law is based upon experimental observation

*The vector d**B** is perpendicular to both d**s** and to the unit vector **r** directed toward the point **P**.*

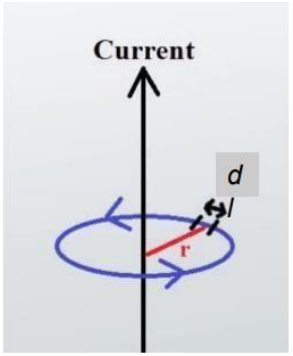
*The magnitude of d**B** is proportional to the current **I**, to the length element d**s** and to the sine of the angle between d**s** and **r**. It is also inversely proportional to r^2.*

Obviously we will Integrate over the entire current distribution.

$$\vec{B} = \frac{\mu_o I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$$

Ampere Law

Magnetic field created by an electric current is proportional to the current with constant of proportionality equal to the permeability of free space.

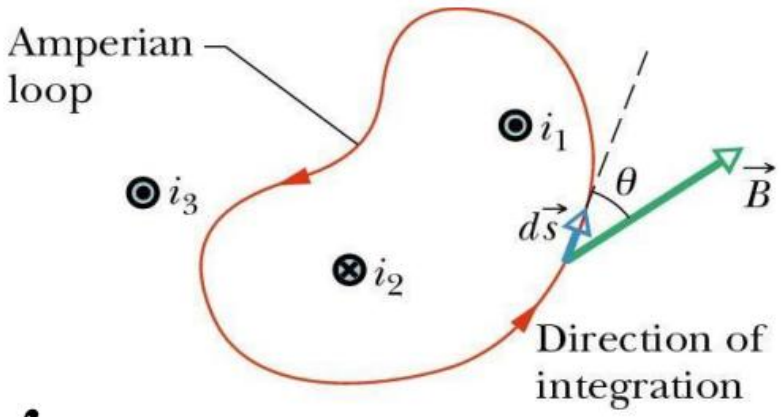


$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enclosed}$$

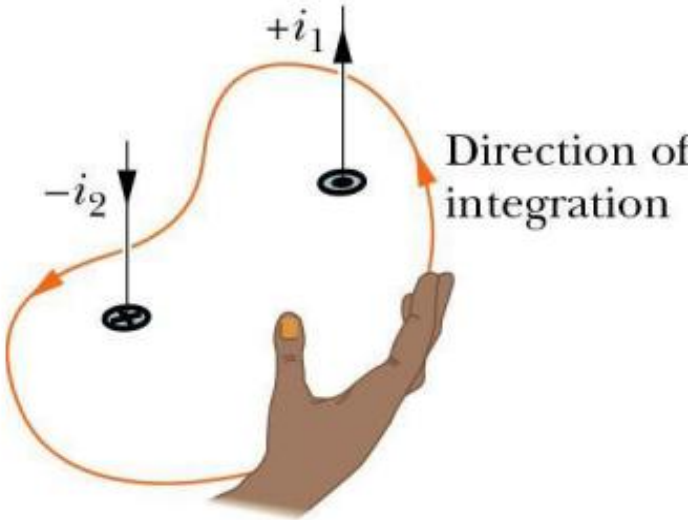
We found for highly symmetric charge distributions, we could calculate the electric field more easily using Gauss's Law than Coulomb's Law. Similarly, Ampère's Law relates the tangential component B_t of the magnetic field summed (integrated) around a closed curve C to the current I_C that passes through any surface bounded by C.

$$\oint \vec{B}\cdot d\vec{r} = \mu_0 I_{\text{encircled}} \quad \text{(Ampère's law, steady currents)}$$

$I_{encircled}$ is the net current that penetrates the surface bounded by the curve C.



$$\oint_C \underline{B}\cdot d\underline{s} = B\cos\theta\, ds = \mu_0 I_{\text{enclosed}}$$



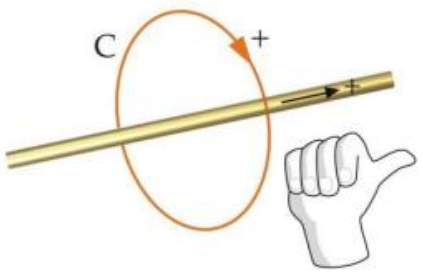
As in Gauss's law, the field that appears in the integral is the net field arising from all sources, not just the encircled currents. We can now use Ampère's Law for situations where symmetry allows us to simplify, and solve the integral for the magnetic field.

$$\oint_C \underline{B} \cdot \underline{dl} = B \oint dl$$

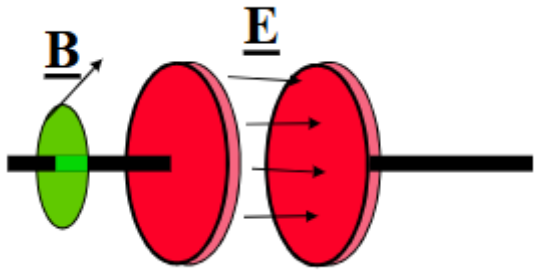
The simplest case is for an infinitely long, straight, current carrying wire. Choose the circular curve centered on the wire. The magnetic field is tangent to this circle from the Biot-Savart law and has the same magnitude around the loop.

$$\oint_C \underline{B} \cdot d\underline{l} = B \oint dl = B 2\pi R = \mu_0 i_{\text{enclosed}}$$

$$B = \frac{\mu_0 i_{\text{enclosed}}}{2\pi R}$$



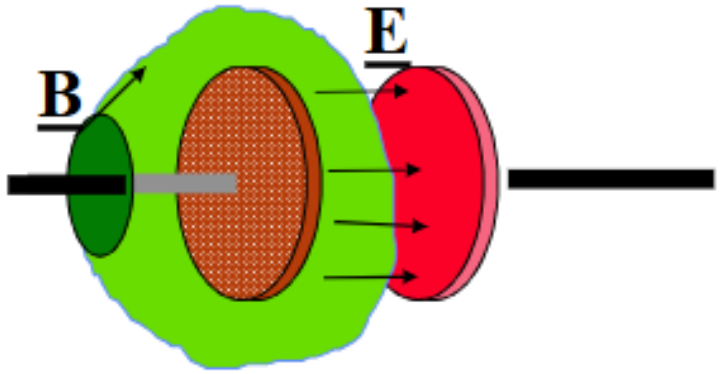
lets take a look at charge flowing into a capacitor



when we derived Ampere's Law we assumed constant current.

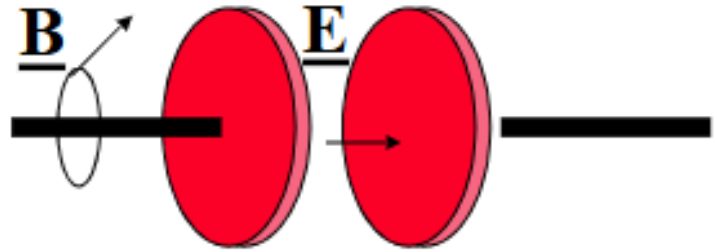
$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I$$

.. if the loop encloses one plate of the capacitor..there is a problem … I = 0

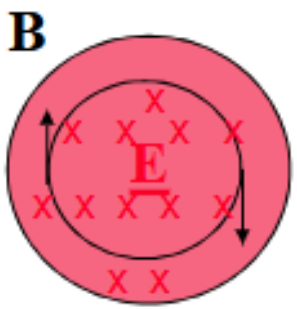


Maxwell solved this problem by realizing that....

Inside the capacitor there must be an induced magnetic field...



How?. Inside the capacitor there is a changing E ⇒



A changing electric field induces a magnetic field

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \mu_0 I_d$$

where I_d is called the ***displacement current***

Therefore, Maxwell's revision of Ampere's Law becomes....

$$\oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Derivation of Displacement Current

For a capacitor, $q = \varepsilon_0 EA$ and $I = \frac{dq}{dt} = \varepsilon_0 \frac{d(EA)}{dt}$.

Now, the electric flux is given by EA, so: $I = \varepsilon_0 \frac{d(\Phi_E)}{dt}$, where this current , not being associated with charges, is called the "Displacement current", I_d.

Hence: $$I_d = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

For a capacitor, $q = \varepsilon_0 EA$ and $I = \frac{dq}{dt} = \varepsilon_0 \frac{d(EA)}{dt}$.

Now, the electric flux is given by EA, so: $I = \varepsilon_0 \frac{d(\Phi_E)}{dt}$, where this current, not being associated with charges, is called the "Displacement Current", I_d.

Hence: $$I_d = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

and: $$\oint \underline{B} \bullet \underline{dl} = \mu_0 (I + I_d)$$

$$\Rightarrow \oint \underline{B} \bullet \underline{dl} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's Equation's in integral form

$$\iint_A \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_o} = \frac{1}{\varepsilon_o} \iiint_V \rho \, dV$$

Gauss's Law of electrostatics

$$\iint_A \vec{B} \cdot d\vec{A} = 0$$

Gauss's Law for Magnetism

$$\int_C \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \iint_A \vec{B} \cdot d\vec{A}$$

Faraday's Law

$$\int_C \vec{B} \cdot d\vec{\ell} = \mu_o I_{encl} + \mu_o \varepsilon_o \frac{d\Phi_E}{dt} = \mu_o \iint_A \left(\vec{J} + \varepsilon_o \frac{d\vec{E}}{dt} \right) \cdot d\vec{A}$$

Ampere's Law

Maxwell's Equations

$$\int_S \mathbf{E} \cdot d\mathbf{A} = \frac{q}{\varepsilon_o}$$

Gauss's law (electricity):

- The total electric flux through any closed surface equals the net charge inside that surface divided by e_o
- This relates an electric field to the charge distribution that creates it

$$\int_S \mathbf{B} \cdot d\mathbf{A} = 0$$

- Gauss's law (magnetism):
- The total magnetic flux through any closed surface is zero
- This says the number of field lines that enter a closed volume must equal the number that leave that volume
- This implies the magnetic field lines cannot begin or end at any point

Maxwell's Equations

$$\int \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt}$$

Faraday's law of Induction:

- This describes the creation of an electric field by a changing magnetic flux
- The law states that the emf, which is the line integral of the electric field around any closed path, equals the rate of change of the magnetic flux through any surface bounded by that path

$$\int \mathbf{B} \cdot d\mathbf{s} = \mu_o I + \varepsilon_o \mu_o \frac{d\Phi_E}{dt}$$

Ampere-Maxwell law is a generalization of Ampere's law

- It describes the creation of a magnetic field by an electric field and electric currents

Maxwell's Equation's in free space

Since there is no charge or current, the general Maxwell's equations reduce to

$$\iint_A \vec{E} \cdot d\vec{A} = 0$$

$$\iint_A \vec{B} \cdot d\vec{A} = 0$$

$$\int_C \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint_A \vec{B} \cdot d\vec{A}$$

$$\int_C \vec{B} \cdot d\vec{\ell} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt} = \mu_o \varepsilon_o \frac{d}{dt}\iint_A \vec{E} \cdot d\vec{A}$$

Maxwell's Equations

Name	Integral Form	Differential Form
Gauss's Law	$\oint_S \vec{E} \bullet d\vec{A} = \frac{Q}{\varepsilon_o}$	$\nabla \bullet \vec{E} = \frac{\rho}{\varepsilon_o}$
Gauss's Law	$\oint_S \vec{B} \bullet d\vec{A} = 0$	$\nabla \bullet \vec{B} = 0$
Faraday's Law	$\oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_B}{dt}$	$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
Ampere-Maxwell's Law	$\oint \vec{B} \bullet d\vec{s} = \mu_o i + \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$	$\nabla \times \vec{B} = \mu_o \vec{J} + \mu_o \varepsilon_o \frac{\partial \vec{E}}{\partial t}$

Electromagnetic wave in free space

Characteristics of electromagnetic wave in dielectric or free space

The electromagnetic waves are transverse in nature .i.e. the electric and magnetic field vector are perpendicular to each other and also perpendicular to the direction of propagation of electromagnetic wave.

There is no phase difference between electric field vector and magnetic field vector in the electromagnetic wave. Both E and B are in phase with each other in electromagnetic wave.

The speed of propagation of the electromagnetic wave in the medium of permittivity (ε) and permeability (μ) is given by the expression. $v = \frac{1}{\sqrt{\epsilon\mu}}$

For free space: $c = \frac{1}{\sqrt{\mu_o \varepsilon_0}} = 2.99 \times 10^8 m/\mathrm{s}$

The electric vector is responsible for the optical effects of an electromagnetic wave and is called ‘light vector’.

The electromagnetic wave being uncharged are not deflected by electric and magnetic fields

The electromagnetic waves are produced by accelerating or oscillating charge.

These waves do not require any material medium for its propagation.

Poynting vector

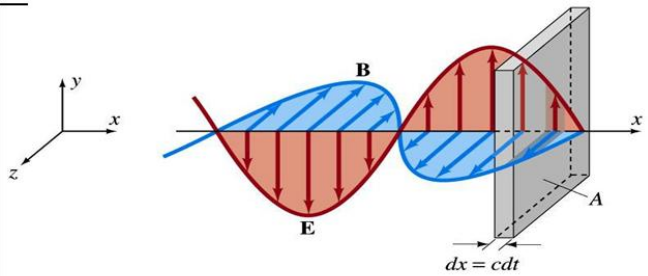
The accelerated electric charge generate electric and magnetic disturbance that can travel infinitely through space and transport energy from one point to other point. The energy transmission in electromagnetic waves has the direction same as the direction of propagation of electromagnetic waves. The rate of energy flow per unit area in a plane electromagnetic wave can be described by a vector called pointing vector

$$\vec{S} = \vec{E} \times \vec{H}$$

Units of pointing vector

The electric field is equal to the potential gradient

i.e. $\vec{E} = \frac{\partial V}{\partial x}$



Unit of E is volt/meter (V/m) and magnetic field (H) is equal to gradient of current

i.e. $(\frac{\partial I}{\partial x})$ its unit is ampere/meter (A/m). Now the unit of pointing vector $\vec{S}$ is

$$\frac{Volt}{meter} \times \frac{Amp.}{meter} = \frac{VA}{m^2}$$

$Volt \times amp$ = electric power measured in watt

S = watt/meter = Wm^{-2} or $J\ s^{-1}\ m^{-2}$

Wave guide

Waveguide are constituted to propagate electromagnetic wave and transfer both power and communication signals.

Depending upon the frequency of electromagnetic wave to be propagated, wave guide can be constructed from either conductive or dielectric materials.

Waveguide consist of a hollow conducting pipe of arbitrary cross section uniform along its length.

Wave guide are especially useful in the propagation of the electromagnetic wave in microwave and optical frequency range.

In waveguide electromagnetic wave are repeatedly reflected between opposite walls and travel down the wave guide in a zig-zag path.

Co-axial cable

It is an electrical cable consisting of an inner conductor surrounded by an insulator and outer cylindrical conductor. The inner conductor and the outer shield sharing the same axis. Coaxial cables are often used as a transmission line for radio frequency signal. A coaxial cable provides protection of the signal from external electromagnetic interference and effectively guides the signals with low emission along the length of the cable.

Coaxial cable consists of the following components

Flexible coaxial cable Outer plastic sheath

Copper screen

Copper core

